Tutorial 7
Advanced Graph Theory

September 9, 2013

1. Determine the stable matchings resulting from the Proposal Algorithm run with men proposing and with women proposing, given the preference lists below.

Men $\{$**u,v,w,x,y,z**$\}$	**Women** $\{$**a,b,c,d,e,f**$\}$
u: $a > b > d > c > f > e$	a: $z > x > y > u > v > w$
v: $a > b > c > f > e > d$	b: $y > z > w > x > v > u$
w: $c > b > d > a > f > e$	c: $v > x > w > y > u > z$
x: $c > a > d > b > e > f$	d: $w > y > u > x > z > v$
y: $c > d > a > b > f > e$	e: $u > v > x > w > y > z$
z: $d > e > f > c > b > a$	f: $u > w > x > v > z > y$

2. Find a minimum weighted cover in the matrix below.

$$\begin{pmatrix} 4 & 5 & 8 & 10 & 11 \\ 7 & 6 & 5 & 7 & 4 \\ 8 & 5 & 12 & 9 & 6 \\ 6 & 6 & 13 & 10 & 7 \\ 4 & 5 & 7 & 9 & 8 \end{pmatrix}$$

3. Prove that if a graph G decomposes into 1-factors, then G has no cut vertex.
4. Prove that a tree T has a perfect matching if and only if $o(T - v) = 1$ for every $v \in V(T)$.
5. Let G be a k-regular graph of even order that remains connected when any $k - 2$ edges are deleted. Prove that G has a $1 - \mathit{factor}$.